

Model-Free Methodology



Volatility Protocol

v1.0 Limited Beta
May 17, 2021

Table Of Contents

[Table Of Contents](#)

[Overview](#)

[Step 1: Calculating Time to Expiration](#)

[Step 2: Selecting Options To Use](#)

[Step 2a: Determine F](#)

[Step 2b: Determine Fstrike](#)

[Step 2c: Select Call and Put Options](#)

[Step 3: Calculate Volatility for Near-Term and Next-Term](#)

[Step 3a: Determine Delta K](#)

[Step 3b: Contribution by Strike](#)

[Step 3c: Contribution by All Strikes](#)

[Step 3d: Calculate Sigma Squared](#)

[Step 3e: Final Step & Calculating ivolETH](#)

[Appendix A - Complete Deribit Data Used In Calculation](#)

Overview

The Model-Free Methodology tracks the expected 14-day volatility for any token for which liquid options markets exist (e.g. ETH & BTC). It uses real-time out of the money (OTM) weekly call and put options data. For the near-term weeklies the expiry is no less than 5 days and no more than 13 days. For the next-term weeklies the expiry is no less than 15 days and no more than 23 days. Origin of the options data is noted for each instrument in the protocol (e.g. ETH options data comes from Deribit).

For each volatility feed an inverse volatility feed is also created whose symbol is denoted by prepending an “i” to the symbol for the original index (e.g. volETH & ivolETH respectively). When volatility moves up the inverse feed moves down by an equivalent percentage, and vice versa. This document uses ETH as an example and shows how the volatility feed for ETH and its inverse are created. We calculate volETH using a similar methodology to that which underpins the VIX® Index¹, the premier volatility benchmark for the U.S. stock market.

¹ Important Note: VIX® Index is a registered trademark of Cboe Global Indices, LLC, used for comparative purposes only. Volatility Protocol is not associated with or endorsed by Cboe or any of its affiliates.

Step 1: Calculating Time to Expiration

$$T = (\text{SettlementTime} - \text{CurrentTime}) / \text{MillisecondsInYear}$$

SettlementTime The UTC date time in milliseconds of the expiration time of the options contract. This is expressed in a UINT256. For example the expiration time on Deribit of February 12th, 2021 8:00 UTC would be expressed as 1613116800000 milliseconds. For T_1 use the Near-Term Expiration. For T_2 use the Next-Term Expiration.
CurrentTime The UTC date time in milliseconds at the time of calculating the index. This is expressed in a UINT245. For example if the current time is February 1st, 2021 18:09 UTC it is expressed as 1612202940000 milliseconds.
MillisecondsInYear 31536000000
Near-Term Expiration The options chain that expires on Friday 8:00 UTC and that is no less than 5 days and no more than 13 days from current time.
Next-Term Expiration The options chain that expires on Friday 8:00 UTC and that is no less than 15 days and no more than 23 days from current time.

Using February 1st, 2021 18:09 UTC as the current time, February 12th, 2021 8:00 UTC for the Near-Term expiration, and February 19th, 2021 8:00 UTC for the Next-Term expiration, T_1 and T_2 are calculated as follows.

$$T = (\text{SettlementTime} - \text{CurrentTime}) / \text{MillisecondsInYear}$$

$$T_1 = (1613116800000 - 1612202940000) / 31536000000$$

$$T_1 = 0.0289783105$$

$$T_2 = (1613721600000 - 1612202940000) / 31536000000$$

$$T_2 = 0.04815639269$$

Step 2: Selecting Options To Use

Step 2a: Determine F

$$F = \text{Strike Price} + e^{RT} \times (\text{Call Price} - \text{Put Price})$$

Strike Price For the Call and Put option orderbooks get the midpoint value between the bid and the ask. If the bid is 0 then return null for that value. If there is a valid bid and no valid ask, then the Mark is used. If the Mid is 1.5x greater than the Mark, then the Mark is used. Take the absolute value of the difference of the Call Mid and the Put Mid. Strike Price is the strike of the smallest Difference value.
e Euler's Number
R This is AAVE's real-time lending interest rate for ETH. For this example use 0.0056
T Time to expiration
Call Price The Call Mid at Strike Price.
Put Price The Put Mid at Strike Price.

Near Term Options

Strike	Call Mid	Put Mid	Difference
1120	null	27.855	null
1200	187.365	49.73	137.635
1280	131.955	81.555	50.4
1360	95.485	125.66	30.175
1440	69.955	null	null
1520	52.05	null	null
1600	38.79	311.055	272.265

Next Term Options

Strike	Cal Mid	Put Mid	Difference
1120	null	51.92	null
1200	null	80.185	null
1280	173.665	116.445	57.22
1360	135.78	163.73	27.95
1440	109.125	null	null
1520	88.16	null	null
1600	71.53	null	null

F calculation based on the tables above.

$$F_1 = 1360 + e^{(0.0056 \times 0.0289783105)} \times (95.485 - 125.66)$$

$$F_1 = 1329.820103$$

$$F_2 = 1360 + e^{(0.0056 \times 0.0481563956)} \times (135.78 - 163.73)$$

$$F_2 = 1332.042462$$

Step 2b: Determine Fstrike

Fstrike is the strike price equal to or directly below F. For this example:

$$F_1 = 1329.820103 \quad \text{then} \quad F \text{ strike}_1 = 1280$$

$$F_2 = 1332.042461 \quad \text{then} \quad F \text{ strike}_2 = 1280$$

Step 2c: Select Call and Put Options

For both the Near-Term and Next-Term options do the following:

Select the Put Mid of OTM put options with strike prices < Fstrike. If a strike has a 0 or null for the Put Mid throw it out.

Strike	Call Mid	Put Mid	Mid Point Selected
1120	null	27.855	27.855
1200	187.365	49.73	49.73
1280			

Select the Call Mid of OTM call options with strike prices > Fstrike. If a strike has a 0 or null for the Call Mid throw it out.

Strike	Call Mid	Put Mid	Mid Point Selected
1280			
1360	95.485	125.66	95.485
1440	69.955	null	69.955
1520	52.05	null	52.05
1600	38.79	311.055	38.79

Average and select the Call Mid and Put Mid for strike price = Fstrike.

Strike	Call Mid	Put Mid	Mid Point Selected
1280	131.955	81.555	106.755

Step 3: Calculate Volatility for Near-Term and Next-Term

Step 3a: Determine Delta K

At the upper and lower bound of any options set, ΔK is the difference between the strike and the adjacent strike.

Near-Term Options lower bound

Strike:	Delta Strike
800	80
880	

Lower bound Delta Strike calculation is $880 - 800$.

Near-Term Options upper bound

Strike:	Delta Strike
2560	
2640	80

Upper bound Delta Strike calculation is $2640 - 2560$.

For all other strikes the ΔK is half the difference of the strikes above and below the target strike.

Near-Term Options

Strike:	Delta Strike:
1120	
1200	80
1280	

Delta Strike calculation is $(1280 - 1120) / 2$

Step 3b: Contribution by Strike

$$\frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i)$$

ΔK_i	The delta strike for i strike.
K_i^2	The strike squared for i strike.
e	Euler's Number
R	This is AAVE's real-time lending interest rate for ETH. For this example use 0.0056
T	Time to expiration
$Q(K_i)$	The selected Mid-Point for the strike.

An example using the 1120 Put from the Near-Term options.

Strike	Call Mid	Put Mid	Delta Strike	Mid Point Selected	Contribution
1120	null	27.855	80	27.855	0.001776755143

$$\frac{\Delta K_{1120Put}}{K_{1120Put}^2} e^{RT} Q(K_{1120Put}) = \frac{80}{1120^2} e^{(0.0056 \times 0.0289783105022831)} (27.855)$$

Contribution = 0.001776755143

Step 3c: Contribution by All Strikes

Calculate the contribution by all strikes for Near-Term and Next-Term. Sum the Contribution by Strike and then multiply by $2/T_1$ and $2/T_2$ for Near-Term and Next-Term respectively.

Near-Term

Strike:	Call	Put	Delta Strike	Mid Point Price Near	Contribution by Strike
800	0	2.325	80	2.325	0.000290672166
880	0	3.645	80	3.645	0.0003766106977
960	0	7.625	80	7.625	0.0006619997808
1040	0	14.925	80	14.925	0.001104099275
1120	0	27.855	80	27.855	0.001776755143
1200	187.365	49.73	80	49.73	0.002763226154
1280	131.955	81.555	80	106.755	0.005213492454
1360	95.485	125.66	80	95.485	0.004130644309
1440	69.955	0	80	69.955	0.002699319179
1520	52.05	0	80	52.05	0.001802577815
1600	38.79	311.055	80	38.79	0.001212384228
1680	29.18	0	80	29.18	0.0008272317367
1760	22.22	0	80	22.22	0.0005739567697
1840	16.915	0	80	16.915	0.0003997576835
1920	13.265	0	80	13.265	0.0002879156424
2000	10.61	0	80	10.61	0.0002122344383
2080	8.285	0	80	8.285	0.0001532238274
2160	6.63	0	80	6.63	0.0001137015774
2240	5.305	0	80	5.305	0.00008459599741
2320	3.98	0	80	3.98	0.00005916536743
2400	3.315	0	80	3.315	0.00004604913885
2480	2.655	0	80	2.655	0.00003453994387
2560	1.66	0	80	1.66	0.0000202669605
2640	2.325	0	80	80	0.00002669165896
				Sum of Contributions ==>	0.02487111194
				$\times 2/T_1 ==>$	1.716532918

Next-Term

Strike:	Call:	Put:	Delta Strike	Mid Point Price Next	Contribution by Strike
960	0	18.635	80	18.635	0.00161805782
1040	0	31.94	80	31.94	0.002363063211
1120	0	51.92	80	51.92	0.003312117567
1200	0	80.185	80	80.185	0.004455923715
1280	173.665	116.445	80	145.055	0.007084673979
1360	135.78	163.73	80	135.78	0.005874421346
1440	109.125	0	80	109.125	0.004211204951
1520	88.16	0	80	88.16	0.003053454911
1600	71.53	0	80	71.53	0.002235915391
1680	58.565	0	80	58.565	0.001660453393
1760	48.255	0	80	48.255	0.001246591295
1840	39.94	0	80	39.94	0.0009440163588
1920	33.28	0	80	33.28	0.0007224170143
2000	27.29	0	80	27.29	0.0005459472089
2080	22.96	0	80	22.96	0.000424670721
2160	19.3	0	80	19.3	0.0003310220412
2240	16.31	0	80	16.31	0.0002601147801
2320	13.64	0	80	13.64	0.0002027895195
2400	11.65	0	80	11.65	0.0001618491965
2480	10.315	0	80	10.315	0.0001342063226
2560	8.32	0	80	8.32	0.0001015898926
2640	7.985	0	80	7.985	0.00009167990877
				Sum of Contributions ==>	0.04116372825
				$\times 2/T_2 ==>$	1.709585205

Then calculate the following equation for both terms:

$$\frac{1}{T} \left[\frac{F}{F \text{ strike}} - 1 \right]^2$$

Near Term

$$\frac{1}{T_1} \left[\frac{F_1}{F \text{ strike}} - 1 \right]^2 = \frac{1}{0.0289783} \left[\frac{1329.820103}{1280} - 1 \right]^2 = 0.05227767221$$

Next Term

$$\frac{1}{T_2} \left[\frac{F_2}{F \text{ strike}} - 1 \right]^2 = \frac{1}{0.0481563} \left[\frac{1332.042462}{1280} - 1 \right]^2 = 0.03432746819$$

Step 3d: Calculate Sigma Squared

$$\sigma_1^2 = \text{SumOfContributions}_1 - \frac{1}{T_1} \left[\frac{F_1}{F \text{ strike}} - 1 \right]^2$$

$$\sigma_1^2 = 1.716532918 - 0.05227767221 = 1.664255246$$

$$\sigma_2^2 = \text{SumOfContributions}_2 - \frac{1}{T_2} \left[\frac{F_2}{F \text{ strike}} - 1 \right]^2$$

$$\sigma_2^2 = 1.709585205 - 0.03432746819 = 1.675257737$$

Step 3e: Final Step & Calculating ivoETH

The final step is to take the square root of the 14 day weighted average of both the Near-Term and Next-Term. Then multiply by 100 to get the ETH Index value.

$$100 \times \sqrt{\left\{ T_1 \sigma_1^2 \left[\frac{N_{T_2} - N_{14}}{N_{T_2} - N_{T_1}} \right] + T_2 \sigma_2^2 \left[\frac{N_{14} - N_{T_2}}{N_{T_2} - N_{T_1}} \right] \right\} \times \frac{N_{365}}{N_{14}}}$$

$$N_{T_1}$$

The number of blocks to settlement of the Near-Term option.

$$(SettlementTime - CurrentTime) / 1000 / AvgBlockTime$$

$$N_{T_2}$$

The number of blocks to settlement of the Next-Term option.

$$(SettlementTime - CurrentTime) / 1000 / AvgBlockTime$$

$$N_{14}$$

The number of blocks in fourteen days based on the AvgBlockTime.

$$14 \text{ Days} \times 24 \text{ Hours} \times 60 \text{ Minutes} \times 60 \text{ Seconds} / AvgBlockTime$$

$$N_{365}$$

The number of blocks in a year based on the AvgBlockTime.

$$365 \text{ Days} \times 24 \text{ Hours} \times 60 \text{ Minutes} \times 60 \text{ Seconds} / AvgBlockTime$$

$$100 \times \sqrt{\left\{ T_1 \sigma_1^2 \left[\frac{114219.3141 - 90974.72924}{114219.3141 - 68731.94946} \right] + T_2 \sigma_2^2 \left[\frac{90974.72924 - 114219.3141}{114219.3141 - 68731.94946} \right] \right\} \times \frac{2371766.918}{90974.72924}}$$

After multiplying by 100 round to the nearest 0.01.

$$\text{ETH Volatility} = 1.29267511 \times 100 = 129.27$$

$$\text{Inverse ETH Volatility} = 1/1.29267511 \times 100 = 77.36$$

Appendix A - Complete Deribit Data Used In Calculation

Feb-1-20212		
	Calls	
Strike	Bid	Ask
800	0	0
880	0	0
960	0	0
1040	0	0
1120	0	0
1200	139.94	234.79
1280	127.31	136.6
1360	94.16	96.81
1440	68.3	71.61
1520	49.73	54.37
1600	36.47	41.11
1680	27.19	31.17
1760	20.56	23.88
1840	15.26	18.57
1920	11.94	14.59
2000	9.28	11.94
2080	7.29	9.28
2160	5.97	7.29
2240	4.64	5.97
2320	3.32	4.64
2400	1.99	4.64
2480	1.33	3.98
2560	1.33	1.99
2640	1.33	3.32

Feb-1-2021		
	Puts	
Strike	Bid	Ask
800	1.33	3.32
880	2.65	4.64
960	6.63	8.62
1040	13.93	15.92
1120	26.53	29.18
1200	47.74	51.72
1280	80.23	82.88
1360	121.35	129.97
1440	0	0
1520	0	0
1600	254.68	367.43
1680	0	0
1760	0	0
1840	0	0
1920	0	0
2000	0	0
2080	0	0
2160	0	0
2240	0	0
2320	0	0
2400	0	0
2480	0	0
2560	0	0
2640	0	0

Feb-19-2021		
	Calls	
Strike	Bid	Ask
960	0	0
1040	0	0
1120	0	0
1200	0	0
1280	169.01	178.32
1360	133.78	137.78
1440	107.13	111.12
1520	85.83	90.49
1600	69.2	73.86
1680	56.57	60.56
1760	46.59	49.92
1840	38.61	41.27
1920	31.95	34.61
2000	25.96	28.62
2080	21.96	23.96
2160	17.97	20.63
2240	15.31	17.31
2320	12.64	14.64
2400	10.65	12.65
2480	9.32	11.31
2560	7.32	9.32
2640	7.32	8.65

Feb-19		
	Puts	
Strike	Bid	Ask
960	17.3	19.97
1040	30.61	33.27
1120	50.59	53.25
1200	78.52	81.85
1280	114.45	118.44
1360	159.07	168.39
1440	0	0
1520	0	0
1600	0	0
1680	0	0
1760	0	0
1840	0	0
1920	0	0
2000	0	0
2080	0	0
2160	0	0
2240	0	0
2320	0	0
2400	0	0
2480	0	0
2560	0	0
2640	0	0